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LOCAL HEAT AND MASS TRANSFER FROM RIGID SPHERES

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NOMENCLATURE

- D, diffusion coefficient;
- d, diameter;
- Fs, Frössling number, $(Sh - 2)/Re^{\frac{1}{2}}Sc^{\frac{1}{3}}$;
- k, mass transfer coefficient;
- Re, Reynolds number, dU_∞/ν;
- Sc, Schmidt number, ν/D;
- Sh, Sherwood number, kd/D;
- U, stream velocity;
- $\bar{u}$, time average longitudinal fluctuating velocity.

Greek symbols
- α_t, apparent level of turbulence, fraction, $[(u)^2]^{\frac{1}{2}}/U_\infty$;
- μ, dynamic viscosity;
- ν, kinematic viscosity;
- ρ, density.

Subscript
- ∞, free stream condition far removed from surface;
- $\bar{\ }$, barred quantities refer to average values.

LEE AND Barrow [1] have recently presented a solution of the boundary-layer equations to calculate local heat and mass transfer coefficients from the forward stagnation point of a sphere to the point of separation. The local Sherwood number was integrated over the front half of the sphere to obtain an equation for the average Sherwood number:

$$\overline{Sh}_f = 0.976\, Re^{0.5}Sc^{0.33}. \tag{1}$$

The authors' experimental mass transfer data for naphthalene, Frössling's [2] experimental data for naphthalene, and Xenakis' [3] data for heat transfer to air were integrated to obtain an equation corresponding to equation (1). The resulting equation is:

$$\overline{Sh}_f = 1.02\, Re^{0.5}Sc^{0.33}. \tag{2}$$

The experimental data were also used to develop an equation for the rear half of the sphere. The equation

$$Sh_r = 0.0447\, Re^{0.78}Sc^{0.33} \tag{3}$$

was found to fit the data for the Reynolds number range of 200–200 000. The experimental data of Xenakis was also included with the two sets of data for naphthalene in this part of the study.

There are two questions with respect to the analysis resulting in equations (2) and (3).

1. The Xenakis data represent supercritical flow regime conditions, therefore, these data should not be included in a correlation with subcritical flow data. The naphthalene data represent a Reynolds number range of 136–25 350.

2. Correlations for $Sh - 2$ are observed to result in a more linear relationship with Reynolds number than does Sh on a log–log plot.

Another objection is that the analysis resulting in equation (1) is based upon an experimental pressure distribution for spheres at a Reynolds number of approximately 200 000. This represents the transcritical flow regime and would not be applicable to the subcritical flow data.

Galloway and Sage [4] analyzed the available experimental data for local heat and mass transfer from spheres and developed correlations for the prediction of local coefficients as a function of Reynolds number, intensity of free stream turbulence, and polar angle. These correlations are based upon data in the Prandtl number range of 0·4–6·8 [3, 5–8] and the Schmidt number range of 0·7–1210 [1, 9–11] and for Reynolds numbers less than 150 000. The correlations are of the form:

$$Fs = \frac{\overline{Sh} - 2}{Re^{0.5}Sc^{0.33}} = A\frac{\nu_\infty}{\nu_i}^{0.16} + [B\alpha_t(\alpha_t + C) + D]\, Re^{0.5}Sc^{0.1667}. \tag{4}$$

It is of interest to utilize the Galloway and Sage correlations for comparison with the conclusions of Lee and Barrow because this work represents experimental data over a wide range of Reynolds numbers, Prandtl–Schmidt numbers, and levels of free stream turbulence. The Frössling naphthalene data are included in the data used in obtaining the correlation.

Forward half of sphere

Figure 1 shows the Sherwood number as a function of the

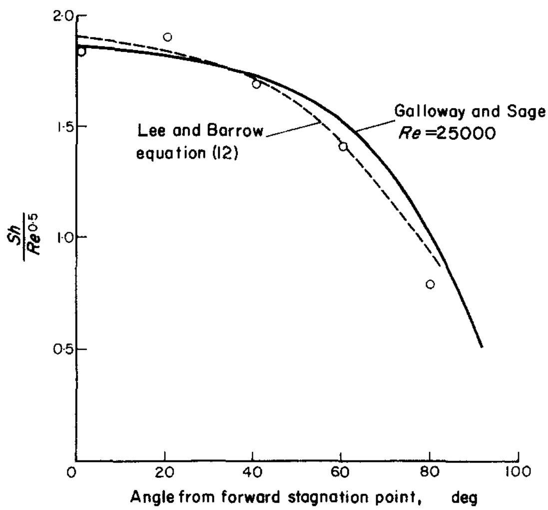

FIG. 1. Comparison of local Sherwood numbers (for $Sc = 2{\cdot}4$).

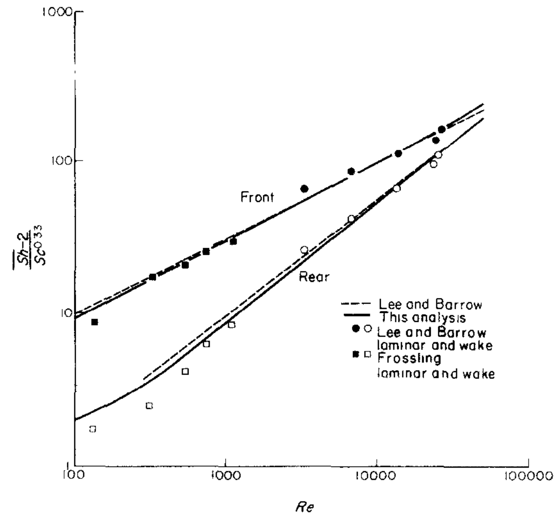

FIG. 2. Average Sherwood numbers for the front and rear hemisphere.

polar angle for Lee and Barrow equation (12) with the experimental data for naphthalene at $Re = 25\,000$. A plot is also shown for $Re = 25\,000$ at zero free stream turbulence as calculated from the Galloway and Sage table of coefficients. This comparison indicates that the data were obtained under conditions of a low level of free stream turbulence. With assumptions of negligible kinematic viscosity gradient and zero free turbulence level, equation (4) reduces to:

$$Fs = \frac{\overline{Sh} - 2}{Re^{0\cdot5}Sc^{0\cdot33}} = A + D\,Re^{0\cdot5}Sc^{0\cdot1667}. \tag{5}$$

The additional assumption that $(A + D\,Re^{0\cdot5}Sc^{0\cdot1667}) =$ constant is necessary to obtain an equation of the form of equations (1) and (2). This is shown by equation (6):

$$Sh_f - 2 = (A + D\,Re^{0\cdot5}Sc^{0\cdot1667})\,Re^{0\cdot5}Sc^{0\cdot33} = KRe^{0\cdot5}Sc^{0\cdot33}. \tag{6}$$

The validity of these assumptions is indicated by comparing values of K from the Lee and Barrow equations with values calculated from the Galloway and Sage correlations.

Table 1.

Source	Re	K
Lee and Barrow—calculated	200 000	0·976
Lee and Barrow—experimental	136–25 000	1·02
Galloway and Sage	100	0·942
Galloway and Sage	25 000	1·06
Galloway and Sage	200 000	1·11

These results indicate the magnitude of the error which results from the assumption that K is constant. It is observed that K for equation (2) is in the range that is predicted by the Galloway and Sage correlations at the experimental Reynolds numbers. Figure 2 shows equation (2) and the $\overline{Sh}_f - 2$ vs. Re relationship calculated from the Galloway and Sage tables. The naphthalene data were recalculated to a $\overline{Sh}_f - 2$ basis and are shown. These data now appear to give a better fit with the Galloway and Sage relationship than with equation (2).

Rear half of sphere

The local Sherwood numbers calculated from the Galloway and Sage coefficients for zero free stream turbulence can also be used for the rear half of the sphere. Average Sherwood numbers can be calculated for equation (5) with the Reynolds and Schmidt numbers as variables to evaluate an equation of the form:

$$\overline{Sh}_r = 2 + a\,Re^b Sc^c. \tag{7}$$

Calculation of a series of values of $\overline{Sh}_r$ with $Sc = 1$ and the Reynolds number range of 100–50 000 gives the results shown by Fig. 2. The equation for the Reynolds number range greater than 500 is:

$$\overline{Sh}_r = 2 + 0{\cdot}035\,Re^{0\cdot8}. \tag{8}$$

Calculation of a series of values of $\overline{Sh}_r$ with constant Reynolds number and a Schmidt number range of 1–1000 indicates an exponent of 0·42 for the Schmidt number. Thus the equation for the rear half of the sphere for a Reynolds number range of 500–50 000 is:

$$\overline{Sh}_r = 2 + 0{\cdot}035\, Re^{0{\cdot}8} Sc^{0{\cdot}42}. \tag{9}$$

Local Sherwood numbers from the Galloway and Sage correlation can also be integrated to obtain the average Sherwood number for the wake region and for the region between 90° and separation. It was found that these two average Sherwood numbers are approximately equal at Reynolds numbers from 100 to 10 000. Thus equation (9) also represents the equation for the wake region. It is interesting to note that the 0·8 exponent for the Reynolds number is the same as is used for heat and mass transfer for turbulent flow in a circular pipe. Similarly, the 0·42 exponent for the Schmidt number is the same as the exponent for experimental heat transfer data with the Prandtl number range of 1–100.

Figure 2 shows equations (3) and (9) with $Sc = 1$. Data for naphthalene are shown as recalculated to $\overline{Sh}_r - 2$ for comparison with the equations and it is observed that the data represent a better fit with equation (9) than with equation (3).

Equation (9) appears to be a general equation that can be used to represent the rear half of a sphere for subcritical flow conditions with the Reynolds number range of 500–50000 and zero free stream turbulence. A general equation of the form of equation (6) for the forward half of the sphere does not appear to be feasible because of the dependence of K on the Reynolds and Prandtl–Schmidt numbers even at zero free stream turbulence.

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